

LIMITS OF ACCURACY

(UPPER/LOWER BOUND)

(3 marks) (P1) ^{original}

$$\begin{array}{l} \text{Accuracy} = 10 \text{ km} \\ \div 2 \\ \hline \text{Error} = 5 \end{array}$$

Distance of Lahore to Islamabad is 400 km to nearest 10 km

Time taken to travel is 4 hours to nearest hour

$$\begin{array}{l} A = 1 \\ \text{Error} = \frac{1}{2} = 0.5 \end{array}$$

(i)

	LOWER BOUND	UPPER BOUND
DISTANCE	$0 - E$ $400 - 5$ <u>395</u>	$0 + E$ $400 + 5$ <u>405</u>
TIME	$0 - E$ $4 - 0.5$ <u>3.5</u>	$0 + E$ $4 + 0.5$ <u>4.5</u>

① $\text{Error} = \frac{\text{Accuracy}}{2}$

② $\begin{array}{l} \text{upper bound} = \text{ORIGINAL} + \text{ERROR} \\ \text{Lower bound} = \text{ORIGINAL} - \text{ERROR} \end{array}$

(ii) Find upper bound of speed { original value not given
accuracy not given } INDIRECT

$$\uparrow s = \frac{d \uparrow}{t \downarrow} = \frac{405}{3.5} = \boxed{}$$

(iii) Find lower bound of speed indirect

$$\downarrow s = \frac{d \downarrow}{t \uparrow} = \frac{395}{4.5} = \boxed{}$$

Advanced:

INDIRECT

Sometimes question will ask for upper bound or lower bound of an indirect value whose original value or accuracy is not given in question.

STEP 1: Write formula for the required quantity.

STEP 2: Analyse the arrows on the equation. First arrow is always given in question.

ADD / SUBTRACT

+ve term = Same arrow

-ve term = opp arrow

$$\uparrow A = \uparrow B + \uparrow C$$

$$\downarrow A = \downarrow B + \downarrow C$$

$$\uparrow A = \uparrow B - \downarrow C$$

$$\downarrow A = \downarrow B - \uparrow C$$

MULTIPLY / DIVIDE

Numerator = Same arrow

Denominator = opp arrow.

$$\uparrow A = \uparrow B \times \uparrow C$$

$$\downarrow A = \downarrow B \times \downarrow C$$

$$\uparrow A = \frac{\uparrow B}{\downarrow C}$$

$$\downarrow A = \frac{\downarrow B}{\uparrow C}$$

orig

Q: Mass of a box is 20 kg to nearest 4 kg $A=4$
 $E=2$

Mass of its packing is 2 kg to nearest $\frac{1}{10}\text{ of a kg}$
 $A=0.1\text{ kg}$
 $E=0.05$

	Lower Bound	Upper Bound
Box	$20-2$ $= 18$	$20+2$ $= 22$
Packing	$2-0.05$ 1.95	$2+0.05$ 2.05

(ii) Find the upper bound of total mass of box and its packing.

Indirect: $\uparrow \text{TOTAL MASS} = \uparrow \text{Box} + \uparrow \text{Packing.}$

$$= 22 + 2.05$$
$$= 24.05\text{ kg.}$$

Ten	10	0.1	Tenth
Hundred	100	0.01	Hundredth
Thousand	1000	0.001	Thousandth

- 2 The cost of a chair, \$x\$, is \$70 ^{orig} correct to the nearest \$10. ^{A=10, E=5}
 The cost of a desk, \$y\$, is \$900 ^{orig} correct to the nearest \$50. ^{A=50, E=25}
- (a) Complete the table in the answer space.

- (b) Find the lower bound of the total cost of a desk and 4 chairs.

Indirect $\downarrow \text{Total} = \downarrow y + 4 \downarrow x$
 $= 875 + 4(65)$
 $= 1135$

Answer (a)

	Lower Bound	Upper Bound
x	65	75
y	875	925

[2]

(b) \$..... [1]

- 6 In a race, an athlete runs 1600 m at an average speed of 6 m/s.
 The distance is given correct to the nearest 100 m and the speed correct to the nearest metre per second.

- (a) Complete the two statements in the answer space.

- (b) Calculate the greatest possible time the race could have taken.

Distance
 orig = 1600
 Acc = 100
 Error = 50

Speed
 orig = 6
 Acc = 1
 E = 0.5

$\uparrow t = \frac{d \uparrow}{s \downarrow} = \frac{1650}{5.5} = 300$

Answer (a) 1550 ≤ distance < 1650
 5.5 ≤ speed < 6.5 [2]

(b)seconds [1]

- 3 (a) The population of a city is given as 280 000, correct to the nearest ten thousand. State the greatest possible error in the given value.

$$\begin{array}{r} A = 10000 \\ \div 2 \\ \hline E = 5000 \end{array}$$

- (b) The dimensions of a rectangular card are 7 cm by 4 cm, correct to the nearest centimetre. Calculate the smallest possible perimeter of the card.

$$\begin{array}{r} A = 1 \\ E = 0.5 \end{array}$$

$$\begin{aligned} \downarrow P &= 2L\downarrow + 2W\downarrow \\ &= 2(6.5) + 2(3.5) \\ &= 13 + 7 \\ &= 20 \end{aligned}$$

Answer (a) 5000 [1]

(b) cm [1]

- 4 (a) A rectangular table top is 100 cm long and 75 cm wide. Both lengths are correct to the nearest 5 cm. Find the least possible perimeter of the table.

$$A = 5, E = 2.5$$

- (b) The area of a rectangular room is 22 m², correct to the nearest square metre. The width is 3 m, correct to the nearest metre. Find the greatest possible length of the room.

$$A = 1, E = 0.5$$

$$A = L \times W$$

$$\begin{aligned} \textcircled{a} \downarrow P &= 2L\downarrow + 2W\downarrow \\ &= 2(97.5) + 2(72.5) \\ &= 340 \end{aligned}$$

$$\begin{aligned} \textcircled{b} \uparrow L &= \frac{A\uparrow}{W\downarrow} \\ &= \frac{22.5}{2.5} \\ &= 9 \end{aligned}$$

Answer (a) cm [2]

(b) m [2]

11 The length of a side of an equilateral triangle is given as 41 mm, correct to the nearest millimetre.

(a) Write down the lower bound for the length of a side.

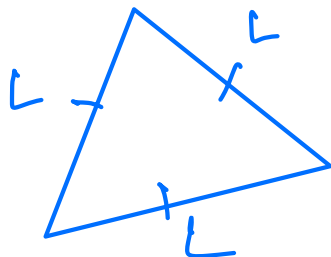
$$A=1$$

$$E=0.5$$

$$40.5$$

Answer (a) mm [1]

(b) Giving your answer in **centimetres**, calculate the lower bound for the perimeter of the triangle.



$$\downarrow P = \downarrow L + \downarrow L + \downarrow L$$

$$= 40.5 + 40.5 + 40.5$$

$$= 121.5 \text{ mm}$$

$$\begin{array}{r} \div 10 \\ \hline 12.15 \end{array}$$

Answer (b) cm [1]

12 ABC is a triangle.

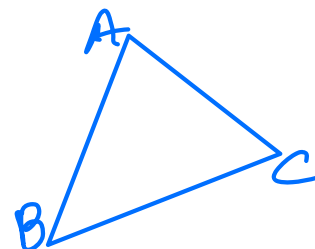
Angle A is 62° , correct to the nearest degree. $A=1, E=0.5$

Angle B is 53.4° , correct to the nearest tenth of a degree.

$$A=0.1, E=0.05$$

(a) Write down the lower bound for angle B .

$$53.4 - 0.05 = 53.35$$



Answer (a) [1]

(b) Calculate the upper bound for angle C .

$$\uparrow C = 180 - \downarrow A - \downarrow B$$

$$= 180 - 61.5 - 53.35$$

$$= \boxed{}$$

Answer (b) [2]